Reverberation-Time Prediction Method for Room Impulse Responses Simulated with the Image-Source Model

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Image-Source Model (ISM)

Background

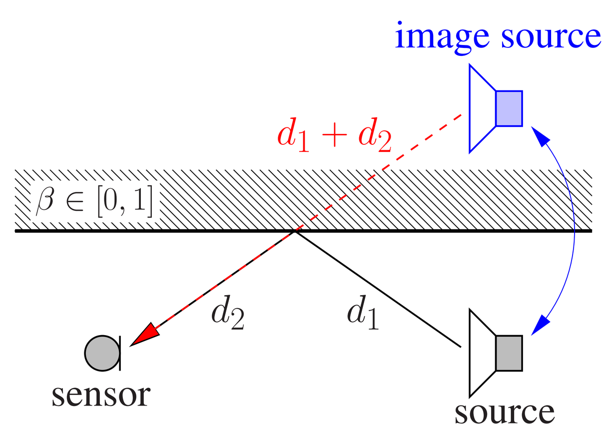
Method originally proposed in: J. Allen and D. Berkeley, "Image method for efficiently simulating small-room acoustics", *J. Acoust. Soc. Am.*, vol. 65(4), 1979.

Aim: simulate "synthetic" room transfer functions (RTFs) between a source and a receiver. Audio signals can be generated through convolution, including scenarios with mobile sound sources.

Advantage: the acoustical properties of the room, e.g., the reverberation time (RT), can be adjusted by varying the walls' sound absorption characteristics.

Basic ISM Principle

- Model the considered environment as a rectangular enclosure with dimensions $\boldsymbol{r} = [L_x, L_y, L_z]^{\mathrm{T}}$.
- Sound source located at $\boldsymbol{p}_s = [x_s, y_s, z_s]^{\mathrm{T}}$ and receiver (acoustic sensor) at $\boldsymbol{p}_r = [x_r, y_r, z_r]^{\mathrm{T}}$.
- Each enclosure surface (walls, floor, ceiling) has a sound reflection coefficient $\beta \in [0, 1]$ (absorption coefficient $\alpha = 1 - \beta^2$).
- Sound reflection off a room boundary is modeled with an image source placed symmetrically across the considered surface:

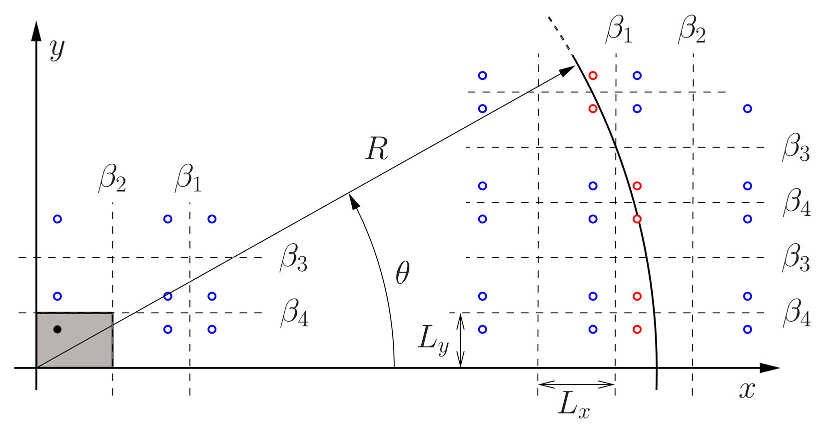


- Attenuation A and time delay τ for a first-order reflection:

$$A = \frac{\beta}{4\pi \cdot (d_1 + d_2)}, \qquad \tau = \frac{d_1 + d_2}{c}.$$

Point-to-Point Room Transfer Function (RTF)

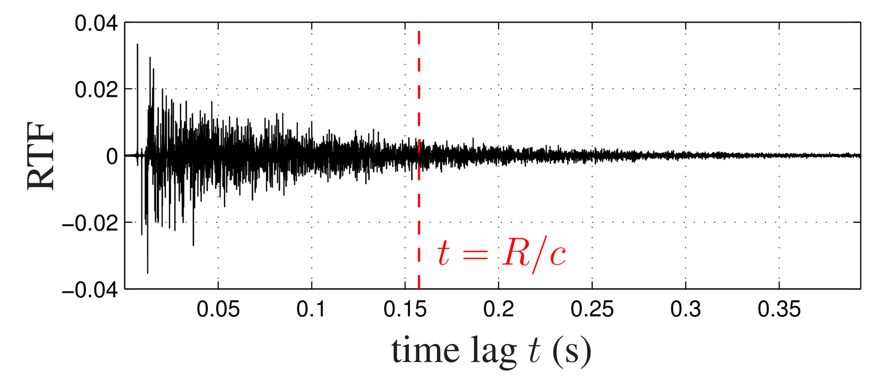
Consider an infinite grid of "image rooms" to model the higher-order reflections. Two-dimensional example:



In the RTF $h(t)$, each image source contributes a Dirac impulse $\delta(\cdot)$ with a specific amplitude $A(\cdot)$ and time delay $\tau(\cdot)$:

$$h(t) = \sum_{\boldsymbol{u}=0}^{1} \sum_{\boldsymbol{v}=-\infty}^{\infty} A(\boldsymbol{u}, \boldsymbol{v}) \cdot \delta\big(t - \tau(\boldsymbol{u}, \boldsymbol{v})\big)$$

with image-source indices $\boldsymbol{u} = (u_x, u_y, u_z)$ and $\boldsymbol{v} = (v_x, v_y, v_z)$.



Energy Decay Curve (EDC)

EDC computed using the normalized Schroeder integration method:

$$E(t) = 10 \cdot \log_{10}\left(\frac{\int_t^{\infty} h^2(\xi)\, \mathrm{d}\xi}{\int_0^{\infty} h^2(\xi)\, \mathrm{d}\xi}\right).$$

EDC Approximation

Proposed Approach

The *energy* RTF $h_E(t)$ at time lag t corresponds to the addition of the energy contribution $a_i(\cdot)$ from any image source i located on a sphere $\mathcal{S}$ of radius $R = c \cdot t$ around the receiver:

$$h_E(t) = \sum_{i \in \mathcal{S}} a_i(R, \theta_i, \varphi_i).$$

Consider the above expression as a Riemann sum representing the integral of a continuous function $a(\cdot)$ over the sphere:

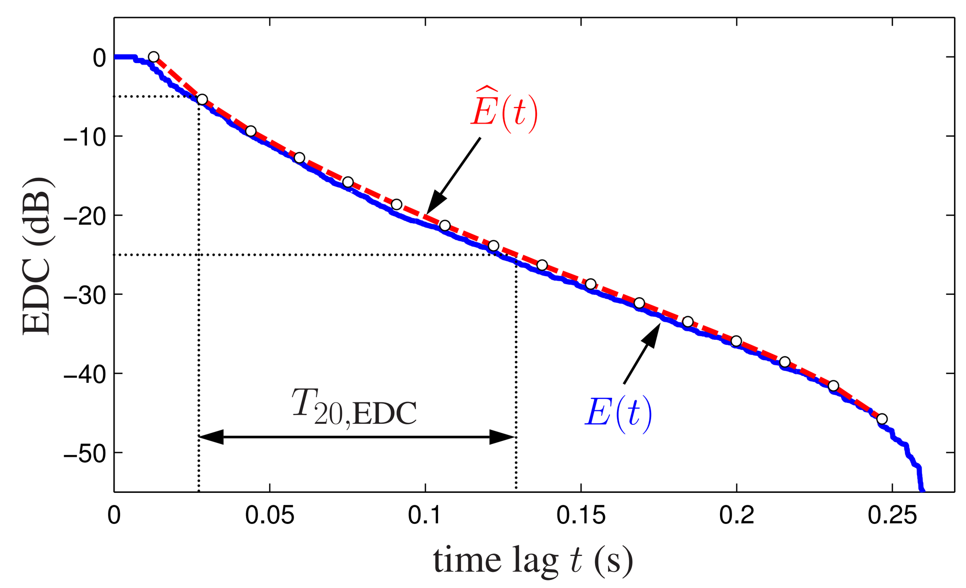
$$\sum_{i \in \mathcal{S}} a_i(R, \theta_i, \varphi_i) \cdot \Delta\varphi\, \Delta\theta \approx \int_0^{2\pi}\!\!\int_0^{\pi} a(R, \theta, \varphi)\, \mathrm{d}\varphi\, \mathrm{d}\theta.$$

Solving the double integral yields an analytical solution for the approximate *energy* RTF (see WASPAA paper for detail):

$$\hat{h}_E(t) = \frac{1}{\Delta\varphi\, \Delta\theta} \cdot \int_0^{2\pi}\!\!\int_0^{\pi} a(R, \theta, \varphi)\, \mathrm{d}\varphi\, \mathrm{d}\theta.$$

The approximate energy decay curve follows as:

$$\widehat{E}(t) \approx 10 \cdot \log_{10}\left(\frac{\sum_{i=0}^{\infty} \hat{h}_E(t + iT)}{\sum_{i=0}^{\infty} \hat{h}_E(t_0 + iT)}\right).$$



RT Prediction

Motivation

Many sound processing algorithms are deployed in reverberant environments. A preliminary assessment of their performance can be achieved with ISM-simulated audio data generated for various reverberation time (RT) values.

⇒ *Testing and comparing different algorithms should be efficient and consistent!*

Typical Approach #1

1. Select a desired RT value, e.g., T_{60}, T_{20}, etc.
2. Determine the walls' reflection coefficients β_k, $k \in \{1, \ldots, 6\}$, from classical RT formulae, e.g., Sabine or Eyring's equations.
3. Simulate the algorithm under test using RTFs from the ISM.
4. Plot the algorithm's performance results vs. the desired RT value.

Known issue: classical RT formulae are inaccurate, especially in conjunction with the ISM ⇒ *Results are potentially flawed!*

Typical Approach #2

1. Select some initial reflection coefficients β_k, $k \in \{1, \ldots, 6\}$.
2. Simulate a few sample RTFs using the ISM, then measure the resulting RT from the computed RTFs.
3. Adjust β_k and repeat step 2 until the desired RT is achieved.

Issue: very time-consuming process, especially for large RT values and/or mobile sound sources (involving many RTFs).

Proposed Approach

Similar to approach #2, but use the proposed EDC approximation method to "predict" $\widehat{E}(t)$ for a given set of β_k, then measure the resulting RT directly from the curve ($T_{20,\mathrm{EDC}}$, see previous graph).

Advantage: fast (no ISM simulation) and accurate process!

Evaluation

How accurately can we predict the RT in ISM-simulated RTFs?

1. Select a desired reverberation time $T_{20,\mathrm{des}}$.
2. Use the proposed RT prediction method to determine the enclosure's absorption coefficients α_k achieving $T_{20,\mathrm{des}}$.
3. Generate a set of RTFs and measure the "real" RT: $T_{20,\mathrm{meas}}$.
4. Compute and plot the resulting error: $\varepsilon = \left|T_{20,\mathrm{des}} - T_{20,\mathrm{meas}}\right|$.

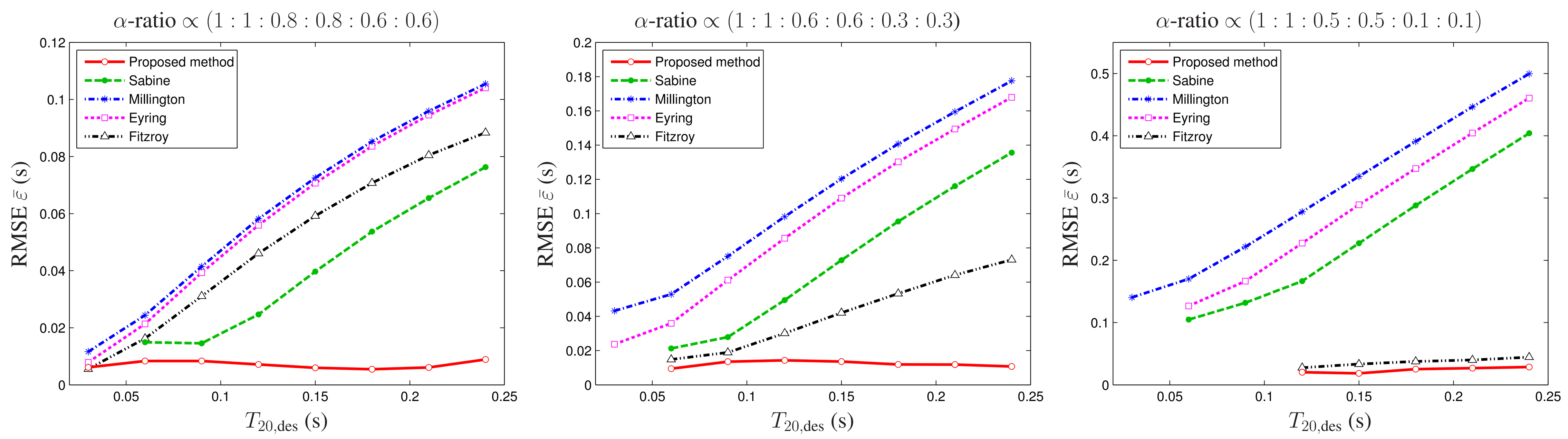
In addition:

- model non-uniform sound absorption using different ratios of α coefficients for each surface: $(\alpha_1 : \alpha_2 : \ldots : \alpha_6)$
- simulate several rooms with different volumes: $20 \ldots 200\mathrm{m}^3$
- test accuracy of classical formulae, e.g., Sabine's RT formula:

$$T_{20,\mathrm{Sab}} = (1/3) \cdot 0.161 \cdot V \Big/ \sum_{k=1}^{6} S_k\, \alpha_k$$

Experimental Results

Root-mean-square error (RMSE) between desired $T_{20,\mathrm{des}}$ and measured RT, with varying absorption coefficient ratios $(\alpha_1 : \alpha_2 : \ldots : \alpha_6)$:



Conclusion

The proposed RT prediction method represents an efficient tool that can be used in conjunction with ISM-based simulations, with the following advantages:

- remains accurate even for a highly non-uniform distribution of sound absorption (outperforms classical RT formulae)
- doesn't require ISM-based RTF simulations (time-consuming)
- provides an efficient control of the RT in simulated RTFs, ensuring consistency when assessing/comparing the performance of audio signal processing algorithms

Other potential domains of interest for the proposed method:

- architectural design
- sound field modeling and auralization
- immersive/interactive virtual environments

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