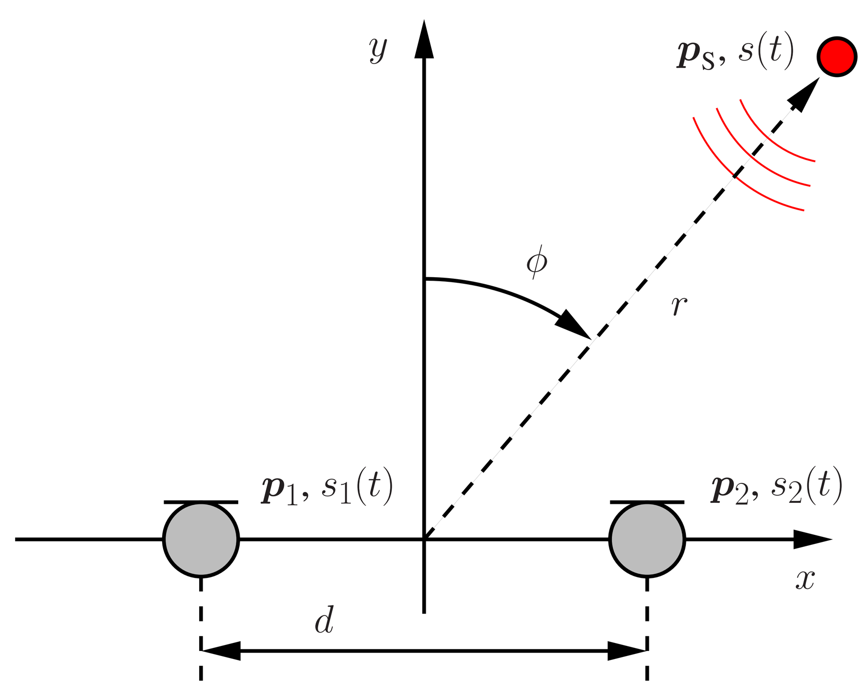
# Particle Filtering Approach to Adaptive Time–Delay Estimation

**Eric A. Lehmann** – WATRI, Perth, Australia

## Time-Delay Estimation (TDE)




$$s_1(t) = s(t) + \eta_1(t)$$
$$s_2(t) = \alpha \cdot s(t - \tau) + \eta_2(t)$$

with time difference of arrival (TDOA) $\tau \in \mathbb{R}$

### Problem Definition

Noisy signals received at two spatially separated sensors:

- known sensor positions $\boldsymbol{p}_i = [x_i \; y_i]^{\mathrm{T}}$, $i = 1, 2$
- unknown source signal $s(\cdot)$
- independent and uncorrelated (diffuse) noise $\eta_1(\cdot)$ and $\eta_2(\cdot)$
- time-varying TDOA $\tau$ (relative source–sensor motion)

### Aim

Online estimate $\hat{\tau}$ of (subsample) TDOA $\tau$ using $s_1(t)$ and $s_2(t)$

## State-State Approach

- State variable at time $k = 1, 2, \ldots$: $X_k$
- Observation variable (measurement): $Y_k$
- Set of observations: $Y_{1:k} = \{Y_1, \ldots, Y_k\}$
- Transition model:

$$X_k = g(X_{k-1}, u_k)$$

- Observation model:

$$Y_k = h(X_k, v_k)$$

where $g(\cdot)$ and $h(\cdot)$ are possibly nonlinear, $u_k$ and $v_k$ are possibly non-Gaussian

### Bayesian Filtering

Compute *posterior* PDF $p(X_k|Y_{1:k})$, then estimate current state:

$$\widehat{X}_k = \int X_k \cdot p(X_k|Y_{1:k}) \, \mathrm{d}X_k$$

### Bayesian Solution

Given $p(X_{k-1}|Y_{1:k-1})$, compute $p(X_k|Y_{1:k})$ recursively:

$$p(X_k|Y_{1:k-1}) = \int p(X_k|X_{k-1}) \, p(X_{k-1}|Y_{1:k-1}) \, \mathrm{d}X_{k-1}$$
$$p(X_k|Y_{1:k}) \propto p(Y_k|X_k) \, p(X_k|Y_{1:k-1})$$

### Problem

Bayesian recursion is only analytical for a small number of practical cases, e.g. Kalman filter (linear and Gaussian problems)

$\Rightarrow$ need for efficient *approximation* techniques!

## Particle Filtering (PF)

- Recursive approximation of the Bayesian filtering solution
- Discrete representation of the posterior PDF with $N$ particles and weights:

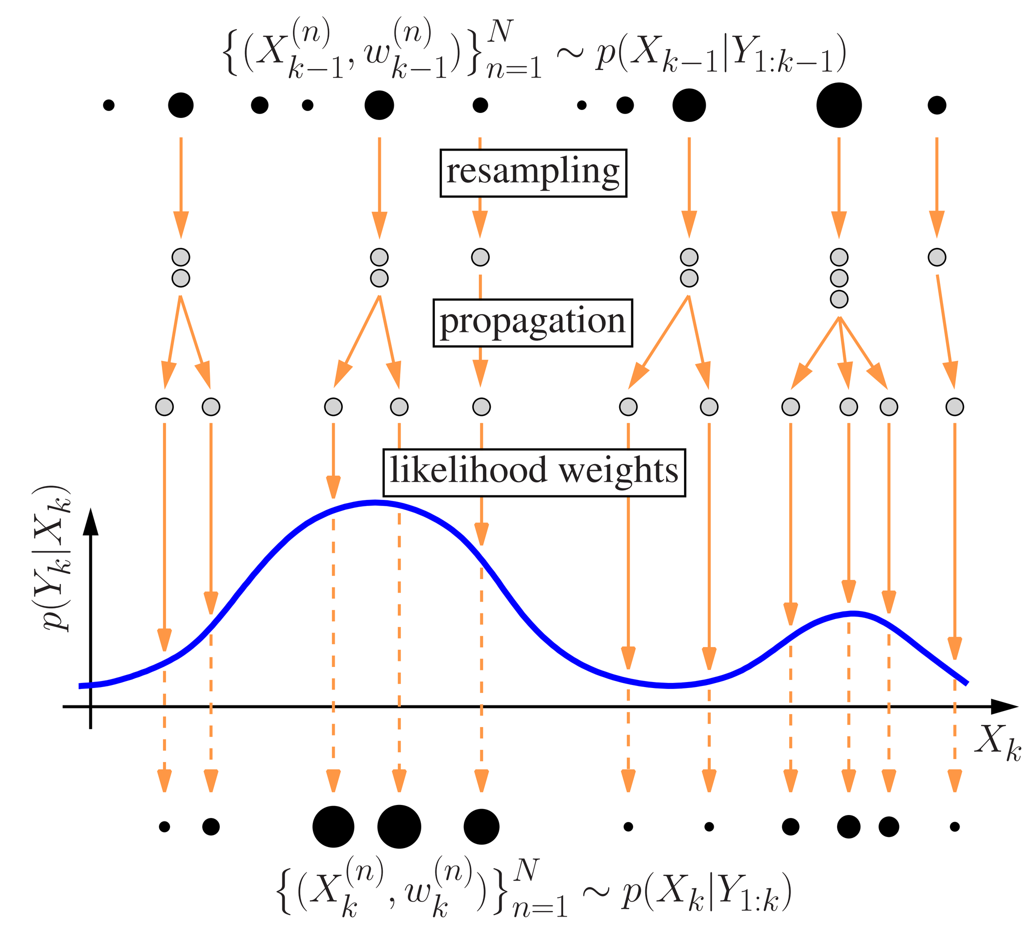
$$\{(X_k^{(n)}, w_k^{(n)})\}_{n=1}^N \sim p(X_k|Y_{1:k})$$

- Estimate of current state:

$$\widehat{X}_k \approx \sum_{n=1}^{N} w_k^{(n)} X_k^{(n)}$$

- Applicable to nonlinear and/or non-Gaussian problems

### PF Recursion




## PF for Time-Delay Estimation

- State variable: $X_k \triangleq \tau_k$
- TDOA modeled as (truncated) FIR fractional-delay filter:

$$\mathbf{h}(k) = \left[h_{-P}(k) \; \cdots \; h_0(k) \; \cdots \; h_P(k)\right]^{\mathrm{T}}$$
$$h_i(k) = \mathrm{sinc}(i - \tau_k)$$

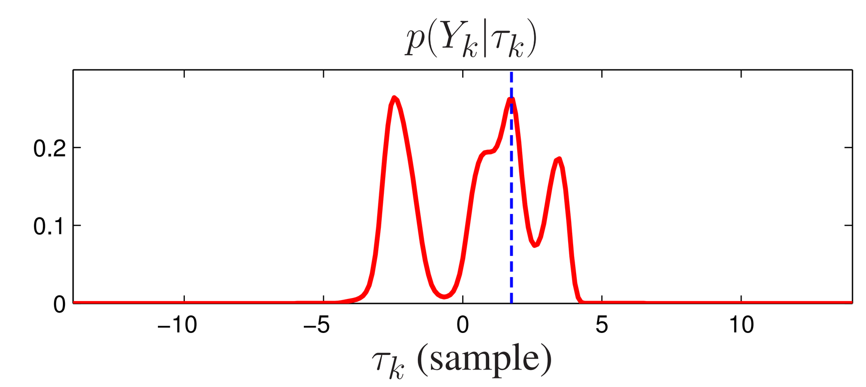
- Signal data collected in overlapping frames, for sensor $i = 1, 2$:

$$\mathbf{s}_i(k) = \left[s_i(k - 2P) \; \cdots \; s_i(k - P) \; \cdots \; s_i(k)\right]^{\mathrm{T}}$$

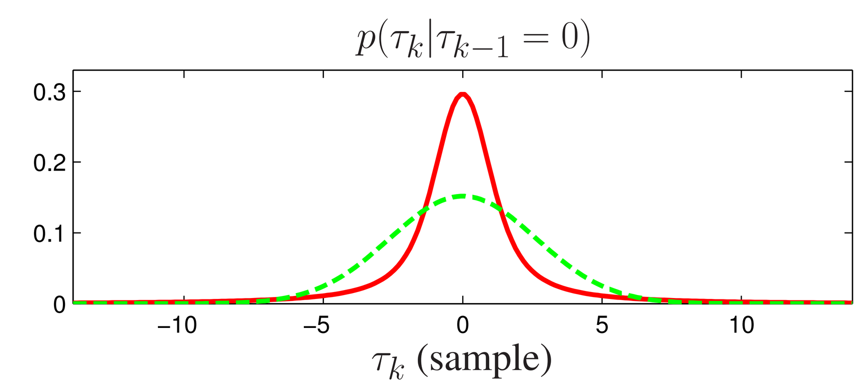
- *Likelihood function.* Observation defined as error between time-shifted signal values:

$$Y_k \triangleq s_2(k - P) - \mathbf{h}(k)^{\mathrm{T}} \mathbf{s}_1(k)$$

Example likelihood function $\Rightarrow$ multi-modal, non-Gaussian




- *Transition PDF.* Source motion modeled as random-walk, near-field or far-field, leads to non-Gaussian transition PDF




## Advantages of Current Approach

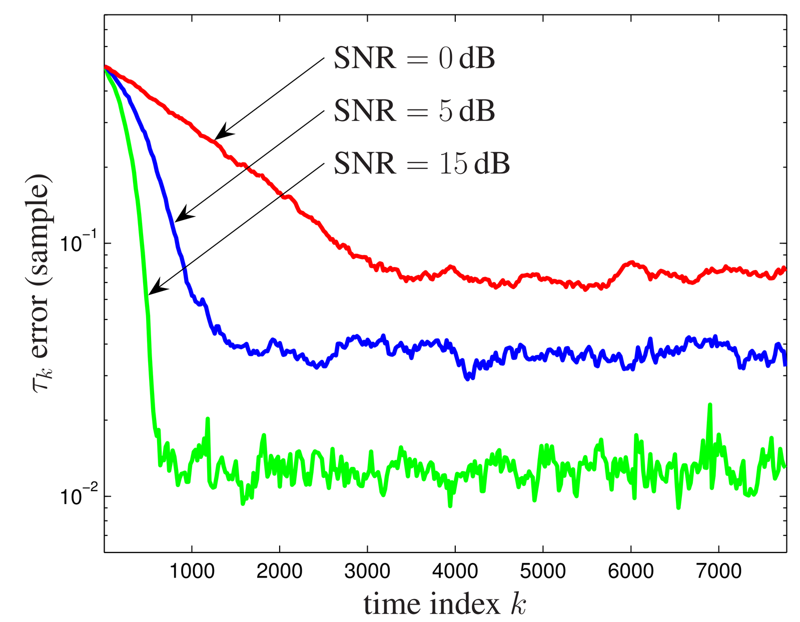
- Includes "dynamics model" of considered target's time delay
- Can represent multi-modal PDFs (multiple hypotheses)
- In comparison to non-adaptive methods (e.g. GCC): no function maximization required, no interpolation to yield subsample time delays

## Experimental Simulation Setup

- Fixed source position, i.e. constant TDOA
- $N = 50$ particles
- FIR filter length $2P + 1 = 51$
- Sampling frequency $f_S = 16\,\mathrm{kHz}$

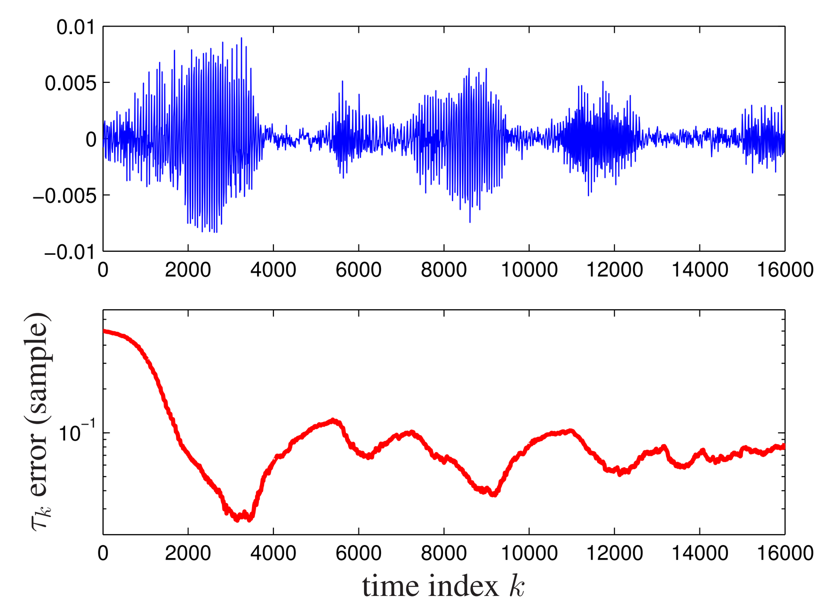
## Simulation Results

PF results using stationary signal (white noise)




## Simulation Results

Non-stationary (speech) signal @ 15 dB SNR




## Conclusions

- PF well suited to handle non-Gaussian problem (e.g. compared to Kalman filter)
- Proposed method based on a sample-by-sample approach
- Preliminary simulation results show that proposed method is able to track correct TDOA with estimation error $< 0.1$ sample
- Method works with both stationary and speech signals

## Future Work

- Assessment with real sound source (moving speaker)
- Assessment in real room situation (reverberation)
- Comparison with other TDE methods (e.g. LMS, ETDE, etc.)
- Include signal attenuation $\alpha$ in state variable